Flipping directing cycles

Kolja Knauer

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Let $G = (V, E)$ be a graph and D an orientation of G. If one reverses any directed cycle in D one obtains an orientation D' with the same out-degree sequence as D, i.e., $\delta^+_D(v) = \delta^+_{D'}(v)$ for all $v \in V$. Conversely, any two orientations D, D' of G with same out-degree sequences can be transformed into each other by reversing a sequence of directed cycles.

Problem 1. Is it true that for any graph G there exists a polynomial-sized set of cycles $\mathcal{F}$, such that if orientations D, D' of G have the same out-degree sequences, then they can be transformed into each other by a sequence of reverses of directed cycles from $\mathcal{F}$?

The answer is known to be *yes* for G of bounded genus or being k-chordal. A first graph that does not fit into any of these classes is Q_n and (for strong orientations of it) a natural candidate for $\mathcal{F}$ would be the set of C_4s. We can't even answer the following:

Problem 2. Is it true that any strongly connected orientation of Q_n contains a directed C_4?